Basic Mathematical Statistics

I Course Description

This course will cover basic concepts in probability and statistics as well as many statistical analysis tools to graduates or advanced undergraduates whose major is not mathematics or statistics. Students enrolled in this course are presumed to know calculus and linear algebra, previous knowledge on probability will be appreciated.

II Contents

1. Probability (12 hrs)

1.1 Sample spaces and events, probability

1.2 Independent events, conditional probability, Bayes' Theorem

1.3 Distribution functions and probability functions, discrete and continuous random variables

1.4 Multivariate distributions, marginal distributions, independent random variables, conditional distributions, transformation of random variables

1.5 Expectation, variance and covariance, conditional expectation, moment generating functions

1.6 Probability inequalities, convergence of random variables, the Law of Large number, the Central Limit Theorem, the Delta Method

2. Statistical Inference (6 hrs)

2.1 Parametric and nonparametric models, Fundamental concepts in inference

2.2 Point estimation, the method of moments, maximum likelihood, Hypothesis testing, multiple testing

3. Statistical Models and Methods (30 hrs)

3.1 Linear and logistic regression, multiple regression, inference about independence

3.2 Directed graphs and conditional independence, undirected graphs, the log-linear model

3.3 Nonparametric curve estimation, kernel density estimation

3.4 Classification, error rates and Bayes classifier, linear classifiers, support vector machines

3.5 Markov chains, Poisson processes, Simulation methods

III Reference Book

1. All of statistics: a concise course in statistical inference. Larry Wasserman, Springer, 2005.

《矩阵理论》教学大纲

一. 概况

1. 开课学院（系）和学科： 理学院 数学系

2. 课程代码：G071555

3. 课程名称：矩阵理论(Matrix Theory)

4. 学时/学分：48 学时/3 学分(每周三学时,共 16 周,第 1 周-第 16 周)

5. 预修课程：线性代数（行列式，矩阵与线性方程组，线性空间 F^n，欧氏空间 R^n，特征值与矩阵的对角化，实对称矩阵与二次型）, 高等数学（一元微积分，空间解析几何，无穷级数，常微分方程）

6. 适合专业：全校的机、电、材、管理、生命和物理、力学诸大学科类，以及人文学科等需要的专业

7. 教材/教学参考书：

《矩阵理论与应用》，张跃辉，科学出版社，2011.

《矩阵理论》， 苏育才、姜翠波、张跃辉编，科学出版社，2006

《矩阵分析》, R.A. Horn and C.R. Johnson, Cambridge Press (中译本)，杨奇译，机械工业出版社，2005。

《矩理阵论与应用》，陈公宁编，高等教育出版社，1990。

《特殊矩阵》，陈景良，陈向晖，清华大学出版社，2001。

《代数特征值问题》，JH.威尔金森著，石钟慈 邓健新 译，科学出版社，2001。

教学团队: 张跃辉, 范金燕, 陈贤锋, 邓大萌, 麻志浩, 陈春丽,邓师瑾

二、课程简介

本课程包含五大部分：线性空间（含内积空间）的结构、线性变换的结构及其与矩阵的关系、矩阵的分解理论及应用、矩阵函数及其微积分、广义逆矩阵与线性方程组的最优解

本课程的核心是线性变换与矩阵分解。课程的主线可以理解为通过线性变换来研究矩阵的结构，赋予矩阵以几何直观，从而更好地运用矩阵的分解理论与微积分理论解决实际问题。

本课程在技术上的重点和难点是矩阵的特征值与矩阵的 Jordan 标准形，因为矩阵计算的实质是特征值的计算，而矩阵的 Jordan 标准形从理论上提供了理解矩阵性质、计算矩阵函数、研究矩阵微积分的一种简便方法。

本课程以研究正规矩阵的分解入手，说明了该类矩阵的分解实际上就是线性变换化为旋转、伸缩、再反转的复合，由此阐明了矩阵分解的框架：即使得相应的线性变换有简明的可操作的几何意义。

本课程的另一个重点是矩阵函数的微积分，特别讨论了线性常微分方程(组)的解和线性系统的可控性与可测性这两个非常重要的应用。

本课程的最后一章讨论矩阵理论比较现代的部分，即广义逆矩阵及其在线性方程组的解或最优解方面的应用。

三、课程的教学内容和要求

矩阵理论的教学内容分为六章，对不同的内容提出不同的教学要求，特别强调应用。

(数字表示供参考的相应的学时数)

第一章　线性代数概要与提高（复习，3）

1．矩阵乘法与分块矩阵　（0.5）

2．线性方程组与 n 维线性空间　（0.5）

3．特征值与矩阵的相似对角化　（0.5）

4．线性空间　（0.5）

5．内积空间与正定二次型　（1）

第一章保留为复习，除了《线性代数》的基本知识外，还包含了一些由线性代数课程可以自然引入的概念和结论，比如矩阵的满秩分解，矩阵的谱和谱半径等等。

要求：掌握矩阵的运算及性质，尤其是对矩阵乘法“左行右列”规则的深入理解和融会贯通；熟练掌握分块初等矩阵以及分块矩阵的运算技巧与要领；领会矩阵应用的思想和方法。

第二章　线性变换与矩阵（9）

1．子空间：直和与空间分解　（2）

要求：理解直和的概念和意义，掌握线性空间分解成线性子空间的直和方法；

2．矩阵与线性变换　（2）

要求：理解线性变换的概念、结构及其意义，能熟练构造线性变换；理解矩阵与线性变换的关系，熟练计算与矩阵相关的四个子空间；

3．内积空间的正交分解　（2）

要求：理解内积空间中子空间的正交补的概念；掌握求矛盾方程组的最小二乘解的理论根据和方法；

4．内积空间中的线性变换　（3）

要求：理解等距变换的定义及与酉矩阵之间的关系，理解正交变换的定义及与正交矩阵之间的关系。

提示与启发：

理解矩阵是坐标化的线性变换，是线性变换在某组(或某对)基下的表达，而线性变换则是最简单也最基本的正比例函数的推广，是若干个多元线性函数。理解对于有限维线性空间，矩阵与线性变换是一回事，但将矩阵理解为线性变换往往更有助于理解问题的实质。

在正交性一节简要介绍了广义相对论所使用的非正定的内积，希望同学能够借此突破传统内积诸如范数平方非负等的“束缚”。本章不仅是矩阵理论的基础，也是学习所有后续数学课程必须具备的预备知识，对本章的深入领会对今后的学习和实践大有裨益。

第三章　特征值与矩阵的 Jordan 标准形(8)

1． Schur 三角化定理：化简矩阵的基础　（2）

要求：掌握 Schur 三角化定理的内容，能够熟练运用该定理化简矩阵；

2． Jordan 标准形：矩阵的一种最简形式　（2）

要求：理解 Jordan 标准形是矩阵在相似意义下的最简形式，理解线性变换的标准形与矩阵的 Jordan 标准形之间的联系；

3． Jordan 标准形的计算　（2）

要求：熟练掌握求幂零矩阵和一般矩阵的 Jordan 标准形的方法和步骤；

4． 盖尔圆定理：特征值的估计　（2）

要求：熟练掌握圆盘定理及其在特征值估计方面的应用；

提示与启发：历史、逻辑和意义（历史与现实、矩阵与几何、矩阵与 IT、矩阵理论与 Nobel 经济学奖）

第四章　正规矩阵与矩阵的分解（12）

1．正规矩阵　（2）

要求：理解正规矩阵的概念和几何意义，熟练掌握实对称矩阵与 Hermite 阵、正交阵与酉阵等正规矩阵；

2．正规矩阵的谱分解　（2）

要求：掌握正规矩阵的谱分解，理解其与线性变换的联系，掌握利用该分解解决实际问题的思想与方法；

3．矩阵的三角分解与 Cholesky 分解　（2）

要求：掌握矩阵的三角分解的方法和意义，掌握计算正定矩阵的 Cholesky 分解的算法；

4．矩阵的 QR 分解　（2）

要求：掌握矩阵的 QR 分解的方法和意义；

5．矩阵的奇异值分解与极分解　（2）

要求：理解矩阵奇异值分解的内容和意义，理解其与线性变换的联系，掌握利用该分解解决实际问题的思想与方法；

6. 应用: 最小二乘法, 图像压缩, 子空间的交 (2)

提示与启发：数学理论（矩阵、线性空间、几何、线性变换）与工程实践（矩阵的分解理论大多产生于工程实践）

第五章 矩阵函数及其微积分（10）

1． 向量与矩阵的范数 （2）

要求： 理解向量和矩阵范数的概念和意义，比较范数和内积诱导的度量的关系；掌握几种基本的向量和矩阵范数及其意义；掌握构造新范数的几种基本方法；

2． 矩阵序列与矩阵级数 （2）

要求：理解矩阵序列与级数，比较矩阵级数与高等数学中的无穷级数之间的异同；

3． 矩阵函数的导数与积分 （1）

要求：理解矩阵函数的定义，掌握 e^{At} 的基本性质，理解矩阵的微积分及其与高等数学中的微积分的异同；

4． 矩阵函数的计算 （1）

要求：掌握利用矩阵的 Jordan 标准形计算矩阵函数的基本方法；

5． 应用 I：线性常微分方程 （2）

要求： 能够利用矩阵函数求线性常系数微分方程组、线性常系数非齐次微分方程组、n 阶常系数微分方程的解；

6． 应用 II：线性系统的可控性与可测性 （2）

要求：能够利用矩阵函数解决定常线性系统的能控性与可观测性问题。

第六章 广义逆矩阵（6）

1．投影矩阵与 Moore-Penrose 广义逆矩阵 （2）

要求： 理解 Moore-Penrose 广义逆 A^+ 的定义及性质，理解广义逆矩阵与投影矩阵的关系；

2. Moore-Penrose 广义逆矩阵的计算 （2）

要求：能够用奇异值分解、满秩分解、谱分解等求 A^+；

3．广义逆 A^- 及广义逆矩阵在线性方程组中的应用 （2）

要求：理解广义逆 A^- 的定义、性质及计算方法；理解 A^+，A^- 与线性方程组的关系；

提示与启发： 历史、逻辑与意义（Moore 广义逆矩阵与 Penrose 广义逆矩阵具有相同的

意义，但遭遇完全不同，为什么？）

四．实验（上机）内容和基本要求

本课程无实验和上机的教学安排，但要求学生结合本专业的特点和所研究的课题，选择部分算法自己上机实现。 要求学生熟悉至少一门数学软件平台（Mathematica/ matleb/Maple）和至少一种编程语言。

五．对学生能力培养的要求

尽管《矩阵理论》是一门具有高度应用价值的课程，但它更是一门研究生的公共数学基础课程，因此将其作为一门理论课比将其作为一门工具课来对待更为恰当。因此教师应在大纲的框架下，引导学生体会课程中出现的数学理论，理解数学公式、定理的推导和证明，使学生掌握将实际问题去粗取精，最终抽象成数学问题（对本课程而言是矩阵问题）的思想与方法，逐步提高学生的数学素质和数学修养，提高学生开展科技活动和社会实践的能力以及开展科研工作的能力。教师应当让每个学生通过解决较为繁琐、复杂甚至艰苦的矩阵、线性空间、线性变换、矩阵的微积分等问题，使每个学生切实体会到科学研究的艰辛和乐趣，培养学生在科学研究和实际工作中顽强拼搏、百折不挠、敢打必胜的信心和意志。

六．其他

最终成绩的评定含期末考试成绩与平时成绩两部分，分别占 70%与 30% 。

起草者：张跃辉

Syllabus for Matrix Theory

1. Information

Course code：	G071555
Course name：	Matrix Theory
Credit hours:	48/3
Semester:	Autumn
Category:	Master Degree Course
Department:	Mathematics
Prerequisite courses:	Advanced Mathematics, Linear Algebra

2. Introduction

Abstract：This course is to introduce students to matrix analysis through the development of essential tools such as linear transformation, the Jordan canonical form, the singular value decomposition and matrix functions and calculus. This course covers both classical and more recent results that are useful in applying matrix algebra to practical problems. In particular it treats linear spaces, linear transformations, eigenvalues and singular values, matrix factorizations, function of matrices and calculus, and generalized inverses of matrices. It builds on the first year linear algebra course. Apart from being used in many areas of mathematics, Matrix Theory has broad applications in fields such as engineering, physics, statistics, econometrics and in modern application areas such as data mining and pattern recognition. Examples from some of these areas will be used to illustrate and motivate some of the theorems developed in the course.

Learning Outcomes

On successful completion of this course students will be familiar enough with matrix theory and linear algebra that they can effectively use the tools and ideas of these fundamental subjects in a variety of applications, understand the importance of spectral decomposition, Schur decomposition, Jordan canonical form and singular value decomposition, understand the role of matrix functions in solving differential equations, understand how to exploit the structure in certain classes of matrices.

3. Contents and Requirements

Chapter 1. Review of linear algebra (3hours)

1．Matrix multiplication and block matrix （0.5）

2．Systems of linear equations and n-dimensional linear spaces （0.5）

3．Eigenvalues and diagonalization of matrices （0.5）

4．Linear spaces （0.5）

5．Inner product spaces and positive definite quadratic forms （1）

Basic requirements: understanding the multiplication rule of matrices. Grasping the operations performed by elementary block matrices, and comprehend the principles of matrix applications.

Chapter 2. Linear Transformation and Matrices (9)

1．Subspace: direct sum and space decomposition （2）

Basic requirements: understanding the concept of direct sum, grasping the method for decomposition of spaces.

2．Matrices and linear transformations （2）

Basic requirements: understanding the concept and the structures of linear transformations, grasping the method for constructing linear transformations；understanding the relationship of matrices and linear transformations, grasping the method for computing the four subspaces associated to a matrix.

3．Orthogonal decomposition of inner product spaces （2）

Basic requirements: understanding the concept of an orthogonal complement, grasping the method for computing the best solutions of a incompatible systems of linear equations.

4．Linear translations over inner product spaces （3）

Basic requirements: understanding the definition of a linear isometry and the relationship between linear isometries and unitary matrices; understanding the definition of orthogonal transformations and the relationship between orthogonal transformations and orthogonal matrices.

Chapter 3. Eigenvalues and Jordan Canonical Forms of Matrices (8)

5． Schur's triangulation theorem （2）

Basic requirements: understanding the meaning of Schur's theorem and grasping the method for simplifying matrices.

6． Jordan canonical form of matrices （2）

Basic requirements: understanding that the Jordan form of a complex matrix is the simplest form of this matrix in the sense of similarity.

7． The Computation of Jordan Canonical Forms （2）

Basic requirements: grasping the method for computing the Jordan forms of nilpotent matrices.

8． Gershgorin theorem （2）

Basic requirements: grasping the method of estimating eigenvalues of matrices via the Gershgorin theorem.

Chapter 4. Eigenvalues and Jordan Canonical Forms of Matrices (12)

1．Normal matrices （2）

Basic requirements: understanding the concept and geometric meaning of normal matrices, mastering real symmetric matrices, Hermitian matrices, orthogonal matrices and unitary matrices.

2．Spectral decomposition of normal matrices （2）

Basic requirements: mastering the spectral decomposition of normal matrices, understanding the correspondence of this decomposition and the related linear transformations，grasping the principle and method of applying this decomposition.

3. Triangular decomposition and Cholesky decomposition of matrices （2）

Basic requirements: understanding the meaning and grasping the method of triangular decomposition of a matrix, grasping the algorithm for computing the Cholesky decomposition of a positive definite matrix.

4. QR decomposition of matrices （2）

Basic requirements: understanding the meaning and grasping the method of the QR decomposition of a matrix.

5. Singular value decomposition and polar decomposition of matrices (2)

Basic requirements: understanding the meaning and grasping the method of SVD of a matrix, understanding the correspondence of this decomposition and the related linear transformations，grasping the principle and method of applying this decomposition.

6. Applications: least squares， image compression， intersection of subspaces (2)

Chapter 5. Matrix Functions and Calculus(10)

7． Vector norms and matrix norms （2）

Basic requirements: understanding the concepts and meanings of vector norms and matrix norms, mastering some frequently-used vector norms and matrix norms, grasping the method of constructing new norms.

8． Matrix sequences and series （2）

Basic requirements: understanding the concepts of matrix sequences and series, comparing the with the same concepts in Advanced Mathematics.

9． Matrix functions and calculus （1）

Basic requirements: understanding the concept of matrix functions, mastering the basic properties of e^{At}, understanding the relationship of matrix calculus and function calculus.

10．Computation of matrix functions （1）

Basic requirements: grasping the method of computing matrix functions via its Jordan canonical form.

11．Application I：systems of linear differential equations （2）

12．Application II：controllability and observability of linear systems（2）

Chapter 4. Generalized Inverses of Matrices（6）

1．Projective matrices and Moore-Penrose inverses of matrices （2）

Basic requirements: understanding the concept of Moore-Penrose inverse A^+, understanding the relationship between Moore-Penrose inverses and projective matrices.

2. Computation of Moore-Penrose inverses （2）

Basic requirements: grasping the method of computing Moore-Penrose inverse via SVD, full rank factorization and spectral decomposition.

3． A^- and applications of generalized inverses to systems of linear equations （2）

Basic requirements: understanding the concept of A^-, understanding the role of A^+ and A^- in solving systems of linear equations.

4. Assessment

Regular assignments: weighting 30%
Two hours end of semester examination: weighting 70%

5. Textbooks and References

1. Matrix Theory and Applications，Yuehui Zhang， Science Press，2011.
2. Matrix Theory, Yucai Su, Cuibo Jiang and Yuehui Zhang, Science Press, 2006.
3. Matrix Analysis, R.A. Horn and C.R. Johnson, Cambridge Press,1985.

4. Matrix Theory and Applications， Gongning Chen, Higher Education Press， 1990.

5. Special Matrices， Jingliang Chen, Xianghui Chen, Tsinghua University Press， 2001.

6. The Algebraic Eigenvalue Problem， J.H. Wilkinson， Oxford University Press， 1965.

计 算 方 法

一、 课程内容

1、课程基本信息

课程代码：G071503

课程名称（中文）：计算方法

课程名称（英文）：Numerical Analysis

学分/学时：3/48

开课时间：秋

课程类别：硕士研究生（非数学专业）数学基础课程

开课院系：理学院数学系

预修课程：高等数学、线性代数、算法语言

面向专业：全校的机、电、材、管理、生命和物理、力学诸大学科类，以及人文学科需要的专业

Basic Information:

Course code：G071503

Course name：Numerical Analysis

Credit hours: 48/3

Semester: Fall

Category: A Master Degree Course

Department: Mathematics

Prerequisite courses: Advanced mathematics, linear algebra, Programming Language

Directions: Majors in Mechanical Engineering, Electrical Engineering, Material Science and Engineering, Economics and Management, Life Science and Biotechnology, Physics, Mechanics or majors in Humanities that are required to take this course.

2、课程内容简介

本课程是非数学类研究生数学公共基础课程，主要讨论用计算机求解数学问题的几类基

本的数值方法及其相关的数学理论。通过本课程的学习，要求学生了解这些数值计算问题的来源，理解求解它们的数学思想和理论根据，数值方法的构造原理及适用范围，掌握相应计算方法及其计算步骤，各种常用的数值计算公式、数值方法的构造原理及适用范围，能够分析计算中产生误差的原因，能采取减少误差的措施；能够解释计算结果的意义，根据计算结果作合理的预测，为今后用计算机去有效地解决实际问题打下基础。

本课程包括数值计算的最基本内容：数值代数，数值逼近，数值积分，方程数值解，常微分方程数值解。

Introduction

This course is a master degree course for all graduate students except those of the Department of Math. Numerical analysis provides the numerical methods and theoretical foundation for a multitude of mathematical problems solving by computer.

The objective of the course includes: 1) know the source of the numerical computation problems, 2) understand the basic idea and the theory of the solving method, the construction principal of the methods, 3) apply the corresponding methods and the formula, 4) analysis the source of the error and know how to control the error, 5) explain the meaning of the numerical results, 6) make prediction based on the numerical results. After the study of this course, the students will gain the basic experience such as to other fields which they may encounter later in the professional career.

This course includes the basic contents of numerical methods: numerical algebra, numerical approximation, numerical integration, numerical solution of equations, numerical solution of ordinary differential equations.

3　教学大纲及学时安排

教学内容分为七部分，对不同的内容提出不同的教学要求

（* 号者为选学部分，视学生接受程度而定）

第一部分　绪论

内容：计算方法的研究目的、特点与基本要求，误差及误差分析等基本概念

要求：了解计算方法在解决实际问题中所处的位置及本课程的内容、研究对象、学习方法、发展简况，理解计算方法中的误差、误差运算及分析、近似计算中应注意的问题、算法的数值稳定性、收敛性与收敛速度等基本概念。

第二部分　插值与逼近

2．1　多项式插值

2．1．1　Lagrange 插值

2．1．2　Newton 插值

2．1．3　Hermite 插值

2．2　分段插值

2．2．1　多项式插值的问题

2．2．2　分段线性插值

2．2．3　分段三次 Hermite 插值

2．3　三次样条插值

2．4　曲线的最小二乘拟合

2．5　最佳平方逼近与正交多项式

*2．6　最佳一致逼近

要求：掌握基本插值法的构造和计算，掌握这些插值函数的余项表达形式、适用范围以及各自特点，了解分段插值及样条插值的特点。理解三次样条函数插值的算法设计。掌握由离散点求曲线拟合的方法，懂得运用最小二乘原理概念以及法方程组进行拟合。掌握正交多项式的概念、基本性质和正交化方法。会使用至少一种正交 Legendre 多项式。在此基础上了解最佳平方逼近与正交多项式的关系。

第三部分　数值积分

3．1　数值积分的基本思想

3．2　Newton-Cotes 公式

3．2．1　Newton-Cotes 公式

3．2．2　复化 Newton-Cotes 公式

3．3　变步长及 Richardson 加速技术

3．4　Gauss 求积法

3．4．1　代数精度

3．4．2　Gauss 形积分公式

3．4．3　Gauss 点

3．4．4　Gauss 形积分公式的特点

要求：掌握常用数值积分法的原理与公式，了解掌握变步长及 Richardson 加速技术，在理解代数精度概念的基础上掌握 Gauss 求积公式及其构造、特点。

第四部分　常微分方程的数值解法

4．1　Eular 法及其变形

4．2　Rung-Kuta 法

4．2．1　泰勒级数法

4．2．2　Rung-Kuta 法的基本思想

4．2．3　二阶 Rung-Kuta 法及其计算公式的推导。

4．2．4　四阶 Rung-Kuta 法

4．3　单步法的收敛性和稳定性

4．4　线性多步法

4．5　方程组与高阶方程的数值解法

要求：掌握理解解常微分方程初值问题的三种构造手段(Taylor 级数法、数值积分法和数值微分法)，会用以上所述方法解常微分方程初值问题，并能对格式作局部截断误差估计。理解单步法的收敛性和稳定性问题的提法和结论。

第五部分　非线性方程求根

5．1　搜索法

5．1．1　逐步搜索法及其特点、适用问题
5．1．2　二分法及其特点、适用问题
5．2　迭代法
5．2．1　迭代法的基本原理
5．2．2　迭代法的收敛与收敛速度
5．3　Newton 法与割线法。

要求：掌握常用的方程求根基本方法，理解这些方法的构造特点及适用范围、对迭代法能进行收敛性、收敛速度分析，理解 Newton 法的特性。

第六部分　线性方程组的数值解法

6．1　Gauss 消去法
6．1．1　Gauss 顺序消去法
6．1．2　Gauss 列主元消去法
6．2　LU 分解方法
6．2．1　LU 分解方法
6．2．2　追赶法、平方根法、LDL 等
6．3　向量与矩阵的范数
6．4　误差分析
6．5　解线性方程组的基本迭代法
6．5．1　Jacobi 迭代法
6．5．2　Gauss-Seidel 迭代法
6．6　迭代法的收敛性
6．7　松弛迭代法

要求：掌握解线性方程组的 Gauss 顺序消元法、列主元素消去法、LU 分解方法，Jacobi 迭代法，Gauss-Seidel 迭代法，及松弛迭代法理解这些方法的构造过程和特点以及适用的线性方程组。掌握迭代算法收敛准则及常用判别条件。了解解特殊线性方程组的追赶法、平方根法、LDL 解法。在掌握向量范数和矩阵范数的基础上了解算法的误差分析及病态方程组概念。

第七部分　矩阵特征值与特征向量的计算

7．1　特征值的定位与估计求矩阵特征值与特征向量的一般原理
7．2　幂法
7．3　QR 分解
7．4　QR 算法

要求：了解求矩阵特征值与特征向量的一般原理，掌握矩阵的 QR 分解，在此基础上了解幂法和 QR 算法的原理和基本算法。掌握用 Householder 变换把矩阵相似约化为上 Hessenberg 阵的算法。

Course Content:

Part I　Introduction

Content:

Motivation, properties and basic requirement of computational methods; fundamental concepts of error and error analysis

Requirement:
Understand the role of computational methods in solving practical problems and the content, research methods, study methods and history of this course.
Understand the error, the operation and analysis of the error in computational methods, the key issues in approximation calculation and the fundamental concepts like the stability of numerical algorithm, convergence and convergence rate.

PART II Interpolation and Approximation

2.1 Polynomial Interpolation
- 2.1.1 Lagrange interpolation
- 2.1.2 Newton interpolation
- 2.1.3 Hermite interpolation

2.2 Piecewise Interpolation
- 2.2.1 Problems in polynomial interpolations
- 2.2.2 Piecewise linear interpolation
- 2.2.3 Piecewise cubic Hermite interpolation

2.3 Cubic Spline Interpolation
2.4 The Least-squares Fitting of Curves
2.5 The Least-squares Approximation and Orthogonal Polynomial
2.6 Best Uniform Approximation (optional)

PART III Numerical Integration

3.1 Basis Ideas in Numerical Integration
3.2 Newton-Cotes Formula
- 3.2.1 Newton-Cotes Formula
- 3.2.2 Compound Newton-Cotes Formula

3.3 Variable Step Size and Richardson Acceleration Method
3.4 Gaussian Quadrature Method
- 3.4.1 Algebra Accuracy
- 3.4.2 Gauss Integral Formula
- 3.4.3 Gauss Point
- 3.4.4 Properties of Gauss Integral Formula

PART IVThe Numerical Solution of Ordinary Differential Equations

4.1 Eular Method and Its Variant
4.2 Runge-Kutta Method
- 4.2.1 Taylor Series Method
- 4.2.2 Basic Idea of Runge-Kutta Method
- 4.2.3 Second-order Runge-Kutta Method
- 4.2.4 Fourth-order Runge-Kutta Method

4.3 Convergence and Stability of Single-step Method
4.4 Linear Multi-step Method
4.5Higher-Order Equations and Systems of Differential Equations

PART V Numerical Methods for Nonlinear Equations

5.1 Search Method
- 5.1.1 Step-by-step Method and Its Properties and Problems in Applications
- 5.1.2 Bisection Method and Its Properties and Problems in Applications

5.2 Iterative Method

5.2.1 Fundamental Principle of Iterative Method

5.2.2 Convergence and Convergence Rate of Iterative Method

5.3 Newton Method and Secant Method

PART VI Solution of Linear Algebraic Equations

6.1 Gauss Elimination Method

6.2 LU Decomposition and Its Applications

6.3 The Norm of Vector and Matrix

6.4 Error Analysis

6.5 Basic Iterative Method

6.5.1 Jacobi Iteration Method

6.5.2 Gauss-Seidel Iteration Method

6.6 Convergence of Iterative Method

6.7 SOR Method

PART VIII Numerical Methods for the Matrix Eigenvalue Problem

7.1 Positioning and Estimating of the Eigenvalue

7.2 Power Iteration Method

7.3 QR Decomposition

7.4 QR Algorithm

二 课程考核要求

1 实验（上机）内容和基本要求

本课程无实验和上机的教学安排，但要求学生结合本专业的特点和所研究的课题，选择部分主要算法自己上机实现。 要求学生熟悉至少一门数学软件平台（Mathematica/Matlab/Maple）和至少一种编程语言。教学实验就是编程解决实际问题。至少做有求解足够规模的问题的大作业 3-4 次，使学生理解如何提出问题和解决问题，以提高分析问题和解决问题的能力。

2 对学生能力培养的要求

本课程以课堂讲授为主，着重讲授算法建立的数学背景、原理和基本线索，教学过程中应该注重方法、概念的理解，注重思维方式培养。每章在介绍各种数值方法正确使用的同时，还要从各种算法的理论分析中了解算法的适应范围且能对一些算法做误差分析，能应用所讲的各种算法在计算机上解决不同的实际问题，使学生建立起自觉使用所学数值方法到本专业中的意识。教师在教学过程中，根据学生的领悟情况，尽量将部分推导演绎过程引导学生自己完成，调动学生动手的欲望，提高授课的质量和效率。

尽管本课程的重点放在运用算法解决问题上，但是仍然鼓励和希望学有余力的同学，对于问题建立模型、算法的性态分析和算法实际运行性质的分析，有实质性的研究和提高。

3 本课程考核的形式以笔试为主，并计入大作业和平时练习的成绩。

三　参考教材与文献

（1）　李庆扬、王能超、易大义，数值分析（第5版），清华大学出版社， 2008

（2）　Timothy Sauer，Numerical Analysis（2nd edition），2011

Numerical Solution of Partial Differential Equations

Instructor:

Name: Dr. Wenjun Ying, School of Mathematical Sciences
Office: Room 503, Bao Yugang Library Building
E-mail: wying@sjtu.edu.cn

Syllabus:

Students of this course should have taken the courses of partial differential equations (PDEs), numerical analysis or their equivalents and at least a course on computer programming with the language of C, C++ or MatLab. This course addresses fundamental concepts, methods and analysis of numerical partial differential equations. It will cover finite difference methods for parabolic and elliptic PDEs, finite element method and boundary integral methods for elliptic PDEs, finite volume method for hyperbolic PDEs of conservation laws.

References:

1. John C. Strikwerda, Finite difference schemes and partial differential equations, second edition, SIAM, 2004.

2. Randall J. LeVeque, Finite difference methods for ordinary and partial differential equations, SIAM, 2007.

3. O. Axelsson and V.A. Barker, Finite element solution of boundary value problems: theory and computation, SIAM, 2001.

4. K. W. Morton and D. F. Mayers, Numerical solution of partial differential equations, second edition, Cambridge, 2005.

5. Dietrich Braess, Finite elements: theory, fast solvers and applications in solid mechanics, Cambridge.

6. Kendall E. Atkinson, The numerical solution of integral equations of the second kind, Cambridge, 1997.

Exams:

There will be one mid-term exam (30% of grade) and a final exam (40% of grade).

Homework:

Homework will be assigned and graded weekly (25% of grade), due the next Friday (in case Friday is on break, due Monday). Homework will be available online at the course webpage approximately one week prior to the due date. Please write your name and section number clearly on each homework set, stapled please! The TA is not responsible for loose sheets of paper that are not stapled together.

Late Policy:

Homework is due at the beginning of class. Homework turned in after the beginning of class will be considered late and will be graded at 80% credit. Late homework will be accepted until 5PM on the due date (no credit thereafter). NO EXCEPTIONS! The policy is intended to keep everyone as current as possible.

Late homework should be given directly to your TA, or placed on your TA's desk, or slipped under your TA's door if the door is locked. Late homework placed anywhere else will not be accepted.

In Class:

You are required to come to class. Should you miss a class, please be sure to get notes and other important information from a classmate. Participation in the classroom discussion is strongly encouraged. Extra bonus points may be given to active discussion participants in the semester grade.

Teaching Plan:

Chapter 1. Introduction (1/2 week)

Chapter 2. Finite difference method for parabolic PDEs (3 weeks)

Section 1. Linear diffusion equation in one space dimension
Energy argument, series solution method
Finite difference methods, including Crank-Nicolson scheme
stability (von Neumann) analysis
Section 2. Reaction-diffusion equation
fully implicit method, semi-implicit method, operator-splitting techniques
Section 3. Linear diffusion equation in multiple space dimensions
Alternating direction implicit method
Locally one-dimensional method

Chapter 3. Finite difference method for elliptic PDEs (3 weeks)

Section 1. Two-point boundary value problem
Maximum principles, finite difference method
Section 2. The Laplace and Poisson equations on rectangles
Maximum principles, finite difference method
The nine-point four-order compact scheme
Section 3. The Laplace and Poisson equations on irregular domains

Chapter 4. Finite element method for elliptic PDEs (4 weeks)

Section 1. Weak derivatives and Sobolev spaces
Section 2. Two-point boundary value problem
Weak form of the problem
Galerkin principle and Ritz principle
Piecewise linear finite element
Error analysis and computer implementation
Section 3. The Laplace and Poisson equation in two space dimensions
Finite elements on rectangles, triangles and quadrilaterals
Error analysis and computer implementation

Chapter 5. Boundary integral method for elliptic PDEs (3 weeks)

Section 1. The delta function and fundamental solution
Section 2. Green's identities
Section 3. The Laplace equation
Section 4. Boundary integrals
Properties of single and double layer boundary integrals and
their normal derivatives on the domain boundary
Section 5. Boundary integral equations
BIEs of the first-kind and second-kind
Section 6. Nystrom and Galerkin methods

Chapter 6. Finite volume method for hyperbolic PDEs (3 weeks)

Section 1. Linear advection equation

The upwind and Lax-Wendroff schemes

Modified equation analysis

Total variation diminishing (TVD) method

Section 2. Scalar nonlinear hyperbolic conservation laws

The Burgers' equation

The method of characteristic lines

The Godunov method

Section 3. The wave equation

Section 4. The shallow water equations

Section 5. The Euler equations

Chapter 7. Conclusion (1/2 week)

最优化方法

一、 基本信息

开课学院（系）：数学科学学院
课程名称：最优化方法
学时/学分：48 学时/3 学分
任课教师：

教材/教学参考书：
1. E. K.P. Chong and S. Zak, An Introduction to Optimization, 4th Edition, Wiley.
2. D. Luenberger and Y. Ye. Linear and nonlinear programing. Springer.
3. Stephen Nash, Ariela Sofer, Linear and Nonlinear Programming
4. D. Bertsekas. Nonlinear programming

Optimization method

Course Information:

This class is intended to cover the central concepts of practical optimization techniques. We introduce the theory and algorithms concerned with finding extrema (maxima and minima) of functions with and without constraints. After a review of topics from calculus and linear algebra, the course offers the students a working knowledge of optimization theory and methods for linear and nonlinear programming, that is, how to find extrema of linear and nonlinear functions subject to various kinds of constraints.

Syllabus:

Chapter 1. Fundamental of optimization
1. Prerequisites from Calculus and Linear Algebra
2. Introduction to optimization models, constraints feasible set, feasible directions
3. Convex sets and functions

Chapter 2. Unconstrained Problems
1. Basic properties of solutions and algorithms
2. Basic gradient methods and line search
3. Newton’s method and variants

4. Conjugate gradient method
5. Quasi-Newton method

Chapter 3. Linear programing

1. Basic Properties of Linear Programs
2. Simplex method
3. Duality
4. Interior point algorithms
5. Transportation and Network Flow Problems

Chapter 4. Constrained minimization

1. Equality and inequality Constrained minimization conditions
2. Lagrange multiplier theory
3. Primal methods: feasible directions methods, active set methods, gradient projection etc.
4. Penalty and Barrier methods
5. Dual methods
6. Primal dual methods